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[Towards optimal softening in 3D codes: I. Minimizing the force error] Towards optimal softening in  
3D *N*-body codes: **I. Minimizing the force error** [Walter Dehnen] Walter Dehnen Max-Planck Institut  
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abstract In simulations of collisionless stellar systems, the forces are softened to reduce the large fluctuations due to shot noise. Softening essentially modifies the law of gravity at  $r = |i - j|$  smaller than some softening length  $\epsilon$ . Therefore, the softened forces are increasingly biased for ever larger  $\epsilon$ , and there is some optimal  $\epsilon$  which yields the best compromise between reducing the fluctuations and introducing a bias. Here, analytical relations are derived for the amplitudes of the bias and the fluctuations in the limit of  $\epsilon$  being much smaller than any physical scale of the underlying stellar system. In particular, it is shown that the fluctuations of the force are generated locally, in contrast to the variations of the potential, which originate from noise in the whole system. Based on these asymptotic relations and using numerical simulations, I study the dependence of the resulting force error on the number of bodies, the softening length, and on the functional form by which Newtonian gravity is replaced. The widely used Plummer softening, where each body is replaced by a Plummer sphere of scale radius  $\epsilon$ , yields significantly larger force errors than do methods in which the bodies are replaced by density kernels of finite extent. I also give special kernels, which reduce the errors even further. These kernels largely compensate the errors made with too small inter-particle forces at  $r < \epsilon$  by exceeding Newtonian forces at  $r \sim \epsilon$ . Additionally, the possibilities of locally adapting  $\epsilon$  and of using unequal weights for the bodies are investigated. By using these various techniques without increasing  $N$ , the rms force error can be reduced by a factor  $\sim 2$  when compared to the standard Plummer softening with constant  $\epsilon$ . The results of this study are directly relevant to simulations using direct summation techniques or the tree code for force evaluation.